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Polarization observables in the processes $p + p \to \Theta^+ + \Sigma^+$ and $n + p \to \Theta^+ + \Lambda^0$, in the threshold region, for any spin and parity of the Θ^+-hyperon

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Abstract. Using the symmetry properties of the strong interaction, such as the Pauli principle, the P-invariance, the conservation of the total angular momentum and isotopic invariance, we establish the spin structure of the threshold matrix elements for the processes $p+p \to \Theta^+ + \Sigma^+$ and $n+p \to \Theta^+ + \Lambda^0$, in the near-threshold region, in a model-independent way, which applies to any spin and parity of the Θ^+-hyperon. We predict the double-spin observables for these processes, such as the dependence of the differential cross-section on the polarizations of the colliding nucleons, and the coefficients of polarization transfer from a nucleon beam or target to the produced Σ^+- or Λ^0-hyperon. We prove that these observables are sensitive to the P-parity of the Θ^+-baryon, for any value of its spin. As an example of dynamical considerations, we analyzed these reactions in the framework of K-meson exchange.

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1 Introduction

The quantum numbers of the possible pentaquark state $\Theta(1540)$ is object of intensive experimental and theoretical considerations. In particular, the P-parity of this baryon is important in order to disentangle different models [1–8].

In principle, specific polarization phenomena in different reactions, such as $\gamma+N \to \Theta^+ + \overline{K}$ [9], $\pi+N \to \Theta^+ + \overline{K}$ and $K + N \to \pi^+ + \Theta^+$ [10,11], $p + p \to \Theta^+ + \Sigma^+$ [12,13], $n + p \to \Theta^+ + \Lambda^0$ [14,15], $p + p \to \pi^+ + \Lambda^0 + \Theta^+$ [9,16] ..., can constitute the tool for an adequate and model-independent way for the determination of the P-parity of the Θ-hyperon. However, the above-listed reactions can be used only if one knows some other quantum numbers of Θ, such as the spin, or, in some cases, the isospin. In general, the mass and the total width do not enter in such considerations. In the reactions involving the K-meson (in initial or in final states) the P-parity of the K-meson has to be known. A typical assumption is that the K-meson is pseudoscalar, following the quark model, but, up to now, there are only indirect experimental indications. We may remind that, in the beginning of photo- and electroproduction studies, both values were considered in the interpretation of the data, [17]. This problem is crucial in the context of the Θ-classification, as the main Θ^+-decay, $\Theta^+ \to NK$, being a strong decay, is controlled by the kaon P-parity.

The purpose of this paper is to generalize the analysis of the determination of the P-parity of the Θ^+-baryon, produced in the simplest reactions of the NN interaction, $p+p \to \Theta^+ + \Sigma^+$ and $n+p \to \Theta^+ + \Lambda^0$, to the case of an arbitrary spin of the Θ^+-hyperon. This analysis can be done in a model-independent form, using the basic symmetry properties of the strong interaction, and does not need any specific dynamical assumptions for the above-mentioned processes. This problem has been analyzed in frame of a different formalism [18].

The central point of this analysis is based on the observation that there is a kinematical region near the reaction threshold, where the final baryons are produced in S-state [19]. This region has an extension in the variable Q ($Q = \sqrt{s} - M_1 - M_2$, s is the square of the total energy of the colliding nucleons, M_1 and M_2 are the masses of the produced hyperons), which is related to the finite radius of the strong interaction. Note that, for the production of strange particles, in NN collisions, such radius is expected to be of the order of $1/m_K$, m_K is the kaon

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mass. Therefore, the corresponding threshold region, with S-wave production, is expected to be quite wide.

This paper is organized as follows. In sect. 2 we consider the production of Θ^+-baryon with $j^P = 3/2^\pm$. We establish the spin structure of the threshold matrix elements for $p + p \to \Theta^+ + \Sigma^+$ and $n + p \to \Theta^+ + \Lambda^0$, and analyze double-spin polarization observables, sensitive to the parity of the Θ^+-hyperon. The generalization on the case of any spin of the Θ^+-hyperon is done in sect. 3.

2 The reaction $\mathbf{N + N \to Y + \Theta^+}$, $\mathbf{Y = \Lambda}$ or $\mathbf{\Sigma}$, $\mathbf{j^P(\Theta^+) = 3/2^\pm}$

The simplest reactions of Θ^+ production in nucleon-nucleon collisions, $p + p \to \Sigma^+ + \Theta^+$ and $n + p \to \Lambda^0 + \Theta^+$, with the lowest threshold energy $E_L = 3.03$ GeV (threshold momentum $p_L = 2.88$ GeV) and 2.82 GeV ($p_L = 2.66$ GeV), respectively, seem good candidates for the determination of the P-parity of the Θ^+-hyperon, through the measurement of different polarization observables. The polarization transfer coefficient from the initial nucleon (beam or target) to the produced Y-hyperon, is relatively easy to measure, because the Λ^0- and the Σ^+-hyperons are self-analyzing particles.

Note, in this respect, that the DISTO Collaboration showed the feasibility of this method, by measuring the D_{nn} coefficient at proton momentum of 3.67 GeV/c, which showed that *"D_{nn} is large and negative ($\simeq -0.4$) over most of the kinematic region"* [20]. It was mentioned in [16] that a nonzero value of D_{nn}, in the threshold region, can be considered as the experimental confirmation of the pseudoscalar nature of the K^+-meson.

Polarization effects in $N + N \to Y + \Theta^+$ for $j^P = 1/2^\pm$, have been studied earlier [12,14,15,13], where it was shown that at least two observables, A_{yy} and D_{yy} are sensitive to the parity of the Θ^+-hyperon, in different ways for $p + p \to \Sigma^+ + \Theta^+$ and $n + p \to \Lambda^0 + \Theta^+$.

A similar derivation will be done for a more complicated case, $j^P = 3/2^\pm$, in the following sections.

2.1 The reaction $\mathbf{n + p \to \Lambda^0 + \Theta^+}$, $\mathbf{j^P(\Theta^+) = 3/2^-}$

The selection rules with respect to the strong interaction allow a single threshold partial transition:

$$S_i = 0, \quad \ell_i = 1 \to \mathcal{J}^P = 1^- \to S_f = 1, \quad \ell_f = 0, \quad (1)$$

where S_i and ℓ_i (S_f and ℓ_f) are the total spin and angular orbital momentum of the initial (final) baryons, $\mathcal{J}^P$ is the total angular momentum and P-parity of the colliding nucleons. We assume, all along this work, that the isotopic spin of the Θ^+-hyperon is equal to zero. This assumption is important only for $n+p \to \Theta^+ + \Lambda^0$ but not for $p+p \to \Theta^+ + \Sigma^+$.

Therefore transition (1) results from the generalized Pauli principle. The corresponding matrix element can be written as

$$\mathcal{M}_\Lambda^{(-)} = f_\Lambda^{(-)} (\tilde{\chi}_2 \sigma_y \chi_1)(\chi_4^\dagger \sigma_y \tilde{\chi}_3^\dagger), \ \chi_4 = \chi_a \hat{k}_a = \boldsymbol{\chi} \cdot \hat{\mathbf{k}}, \ (2)$$

where $\hat{\mathbf{k}}$ is the unit vector along the three-momentum of the colliding nucleons, in the reaction CM system, χ_1 and χ_2 are the two-component spinors of the initial nucleons, χ_3 is the two-component spinor of the produced Λ-hyperon and χ_a is the two-component spinor with three-vector index a, describing the polarization properties of the Θ^+ with spin 3/2, obeying the following condition:

$$\boldsymbol{\sigma} \cdot \boldsymbol{\chi} = 0, \quad (3)$$

and, finally, $f_\Lambda^{(-)}$ is the partial amplitude for the singlet np interaction.

Equation (3) allows to derive the dependence of the differential (and total) cross-section on the polarizations $\mathbf{P}_1$ and $\mathbf{P}_2$ of the colliding nucleons:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(-)}(\mathbf{P}_1, \mathbf{P}_2) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 (1 - \mathbf{P}_1 \cdot \mathbf{P}_2), \quad (4)$$

independently of the amplitude $f_\Lambda^{(-)}$, *i.e.* independently of the concrete dynamics for the considered reaction.

It is straightforward to show that all polarization transfer coefficients, characterizing the dependence of the polarization of any final baryon on the polarization of any initial nucleon, vanish. The physical reason is that the singlet np-state, being "closed" with respect to both nucleon spins, does not contain and does not transmit any information on the polarization to the final baryons.

Therefore the predictions

$$A_{xx} = A_{yy} = A_{zz} = -1,$$
$$D_{xx} = D_{yy} = D_{zz} = 0 \quad (5)$$

are typical properties for negative P-parity Θ^+ production for $n + p \to \Lambda^0 + \Theta^+(3/2^-)$ in the threshold region.

Note that the z-axis is taken along the beam direction, and the axial symmetry of S-wave production results in the equality of (xx) and (yy) components of all polarization observables.

2.2 The reaction $\mathbf{n + p \to \Lambda^0 + \Theta^+}$, $\mathbf{j^P(\Theta^+) = 3/2^+}$

The symmetry selection rules (P-invariance, conservation of the total angular momentum and the validity of the generalized Pauli principle) result in the following three partial transitions:

$$\begin{aligned} S_i = 1, \quad \ell_i = 0 &\to \mathcal{J}^P = 1^+ \to S_f = 1, \quad \ell_f = 0, \\ S_i = 1, \quad \ell_i = 2 &\to \mathcal{J}^P = 1^+ \to S_f = 1, \quad \ell_f = 0, \quad (6) \\ &\to \mathcal{J}^P = 2^+ \to S_f = 2, \quad \ell_f = 0. \end{aligned}$$

The corresponding matrix element can be written as

$$\begin{aligned} \mathcal{M}_\Lambda^{(+)} = & f_{1\Lambda}^{(+)} (\tilde{\chi}_2 \sigma_y \boldsymbol{\sigma} \cdot \hat{\mathbf{k}} \chi_1)(\boldsymbol{\chi}^\dagger \cdot \hat{\mathbf{k}} \sigma_y \tilde{\chi}_3^\dagger) \\ & + f_{2\Lambda}^{(+)} \left[\tilde{\chi}_2 \sigma_y (\sigma_m - \hat{k}_m \boldsymbol{\sigma} \cdot \hat{\mathbf{k}})\chi_1\right] (\chi_m^\dagger \sigma_y \tilde{\chi}_3^\dagger) \quad (7) \\ & + i f_{3\Lambda}^{(+)} \left[\tilde{\chi}_2 \sigma_y (\boldsymbol{\sigma} \times \hat{\mathbf{k}})_m \chi_1\right] (\boldsymbol{\chi}^\dagger \cdot \hat{\mathbf{k}} \sigma_m \sigma_y \tilde{\chi}_3^\dagger), \end{aligned}$$

where $f_{i\Lambda}^{(+)}$, $i = 1, 2, 3$ are the partial amplitudes for the case of positive parity of the Θ^+-hyperon. Note that the amplitude $f_{1\Lambda}^{(+)}$ describes the triplet np interaction in a state with zero value of total spin projection, but the amplitudes $f_{2,3\Lambda}^{(+)}$ describe the np interaction with total spin projection equal to ± 1.

Therefore, the cross-section for polarized $\mathbf{n} + \mathbf{p}$ collisions can be written as

$$\frac{d\sigma}{d\Omega}^{(+)}(\mathbf{P}_1 \cdot \mathbf{P}_2) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 + \mathcal{A}_1^{(+)} \mathbf{P}_1 \cdot \mathbf{P}_2 + \mathcal{A}_2^{(+)} \hat{\mathbf{k}} \cdot \mathbf{P}_1 \hat{\mathbf{k}} \cdot \mathbf{P}_2\right],$$
$$D_\Lambda^{(+)} \mathcal{A}_1^{(+)} = |f_{1\Lambda}^{(+)}|^2,$$
$$D_\Lambda^{(+)} \mathcal{A}_2^{(+)} = 2\left(-|f_{1\Lambda}^{(+)}|^2 + |f_{2\Lambda}^{(+)}|^2 + |f_{3\Lambda}^{(+)}|^2\right), \quad (8)$$

where $D_\Lambda^{(+)} = |f_{1\Lambda}^{(+)}|^2 + 2\left(|f_{2\Lambda}^{(+)}|^2 + |f_{3\Lambda}^{(+)}|^2\right)$.

The relation $3\mathcal{A}_1^{(+)} + \mathcal{A}_2^{(+)} = 1$, which is correct for any values of partial amplitudes $f_{i\Lambda}^{(+)}$, results from the absence of the singlet np interaction —for the considered case.

One can find from (8)

$$A_{xx}^{(+)} = A_{yy}^{(+)} = \frac{1}{2}\left(1 - A_{zz}^{(+)}\right) = \frac{|f_{1\Lambda}^{(+)}|^2}{|f_{1\Lambda}^{(+)}|^2 + 2\left(|f_{2\Lambda}^{(+)}|^2 + |f_{3\Lambda}^{(+)}|^2\right)} \geq 0, \quad (9)$$

which is different from the case of negative P-parity.

The same conclusion holds for the case of the coefficients of polarization transfer. The dependence of the Λ-polarization $\mathbf{P}_\Lambda$ on the initial beam polarization, $\mathbf{P}$ (described by the two-component spinor χ_2) can be written in the following form, which holds for S-wave production:

$$\mathbf{P}_\Lambda = p_1^{(+)} \mathbf{P} + p_2^{(+)} \hat{\mathbf{k}} (\hat{\mathbf{k}} \cdot \mathbf{P}), \quad (10)$$

where $p_{1,2}$ are real coefficients, which can be expressed as a function of the amplitudes as

$$D_\Lambda^{(+)} p_1^{(+)} = 2|f_{2\Lambda}^{(+)}|^2 - \mathrm{Re} f_{1\Lambda}^{(+)} (f_{2\Lambda}^{(+)} + 2 f_{3\Lambda}^{(+)})^*,$$

$$D_\Lambda^{(+)} p_2^{(+)} = -|f_{2\Lambda}^{(+)}|^2 + 2|f_{3\Lambda}^{(+)}|^2 + \mathrm{Re}\left[f_{1\Lambda}^{(+)} f_{2\Lambda}^{(+)*} + 2(f_{1\Lambda}^{(+)} + f_{2\Lambda}^{(+)}) f_{3\Lambda}^{(+)*}\right]. \quad (11)$$

Therefore, generally, the polarization transfer coefficients

$$D_{xx}^{(+)} = D_{yy}^{(+)} = p_1^{(+)}, \ D_{zz}^{(+)} = p_1^{(+)} + p_2^{(+)}$$

do not vanish, in this case.

In order to have a quantitative estimation of these polarization observables, let us consider a simple model for $n + p \to \Lambda + \Theta^+$, based on t-channel K-exchange (fig. 1).

Note that only the coherent sum of the two diagrams, with vertices which satisfy the isotopic invariance, results

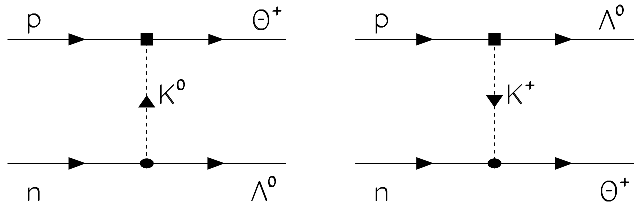

Fig. 1. K-exchange for the reaction $n + p \to \Lambda^0 + \Theta^+$.

in a correct spin structure of the threshold matrix element, see eq. (7). Each diagram generates a spin structure with a singlet amplitude, which cancels in the final sum. The final result for the considered mechanism can be written as

$$f_{1\Lambda}^{(+)} = -f_{3\Lambda}^{(+)}, \ f_{2\Lambda}^{(+)} = 0. \quad (12)$$

Let us stress that these predictions do not depend on the values of the coupling constants $g_{N\Lambda K}$ and $g_{N\Theta K}$, and on the phenomenological form factors which are important ingredients of this model. These quantities enter in the calculation of the absolute values of the differential (and total) cross-section, but not in the predictions for the polarization observables.

Substituting eq. (12) in eqs. (8) and (11), one can find

$$A_{xx}^{(+)} = A_{yy}^{(+)} = A_{zz}^{(+)} = \frac{1}{3},$$
$$D_{xx}^{(+)} = D_{yy}^{(+)} = -\frac{2}{3}, \qquad D_{zz}^{(+)} = \frac{2}{3}. \quad (13)$$

Note that the predicted result $D_{xx}^{(+)} = -2/3$ coincides with $D_{nn}^{(+)} = -2/3$, which can be found for $\vec{p} + p \to \vec{\Lambda} + K^+ + p$ —in the framework of a similar model, based on K-exchange, in agreement with the existing DISTO results. We stress again, that this simple model is taken here only for illustration.

Another confirmation of the validity of this simple model, which takes into account only K-exchange for threshold strange-particle production in pp collisions is the large value of the ratio $\sigma(pp \to \Lambda K^+ p)/\sigma(pp \to \Sigma K^+ p)$, measured in COSY [19].

Considering the similarity of final states in $N + N \to \Lambda + K^+ + N$ and in the reaction of interest here, $n + p \to \Lambda + \Theta^+ \to \Lambda + K^+ + n$, one can assume that the K-exchange model can be applied also here. Of course, this is not a proof, but a qualitative argument to justify the model taken here for a quick estimation of polarization effects in $\Lambda + \Theta^+(3/2^+)$, with a complicated spin structure.

The result found here, $D_{yy}^{(+)} = -2/3$, (which is model dependent in case of $j^P(\Theta^+) = 3/2^+$) is very far from the value $D_{yy}^{(-)} = 0$, which has been found in a model-independent way for the opposite parity of Θ^+, and shows the level of accuracy which will be necessary, in order to discriminate the different parities of the Θ^+-baryon.

2.3 The reaction $\mathbf{p}+\mathbf{p}\to\mathbf{\Sigma}^++\mathbf{\Theta}^+$, $\mathbf{j}^{\mathsf{P}}(\mathbf{\Theta}^+)=3/2^+$

For a positive parity, only one spin transition is allowed at threshold:

$$S_i=0,\quad \ell_i=2\to\mathcal{J}^P=2^+\to S_f=2,\quad \ell_f=0, \tag{14}$$

to which corresponds the following matrix element:

$$\mathcal{M}_{\Sigma}^{(+)}=f_{\Sigma}^{(+)}(\tilde{\chi}_2\sigma_y\chi_1)(\boldsymbol{\chi}^\dagger\cdot\hat{\mathbf{k}}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\sigma_y\tilde{\chi}_3^\dagger). \tag{15}$$

The dependence of the differential cross-section on the polarizations of the initial nucleons is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\mathbf{pp}\to\Sigma^+\Theta^+)=\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0(1-\mathbf{P}_1\cdot\mathbf{P}_2),$$

and the polarization transfer coefficients

$$D_{xx}^{(+)}(\Sigma)=D_{yy}^{(+)}(\Sigma)=D_{zz}^{(+)}(\Sigma)=0, \tag{16}$$

i.e. the result is similar to the reaction $n+p\to\Lambda^0+\Theta^+(3/2^-)$, but with opposite parity!

2.4 The reaction $\mathbf{p}+\mathbf{p}\to\mathbf{\Sigma}^++\mathbf{\Theta}^+$, $\mathbf{j}^{\mathsf{P}}(\mathbf{\Theta}^+)=3/2^-$

The selection rules related to the strong interaction allow the following partial transitions:

$$\begin{aligned}S_i=1,\quad \ell_i=1&\to\mathcal{J}^P=1^-\to S_f=1,\quad \ell_f=0,\\ &\to\mathcal{J}^P=2^-\to S_f=2,\quad \ell_f=0,\\ S_i=1,\quad \ell_i=3&\to\mathcal{J}^P=2^-\to S_f=2,\quad \ell_f=0.\end{aligned} \tag{17}$$

with matrix element

$$\begin{aligned}&\mathcal{M}_{\Sigma}^{(-)}=\\ &f_{1\Sigma}^{(-)}(\tilde{\chi}_2\sigma_y\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\chi_1)(\boldsymbol{\chi}^\dagger\cdot\hat{\mathbf{k}}\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\sigma_y\tilde{\chi}_3^\dagger)\\ &+f_{2\Sigma}^{(-)}\left[\tilde{\chi}_2\sigma_y(\sigma_m-\hat{k}_m\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\chi_1\right](\boldsymbol{\chi}^\dagger\cdot\hat{\mathbf{k}}\sigma_m\sigma_y\tilde{\chi}_3^\dagger)\\ &+if_{3\Sigma}^{(-)}\left[\tilde{\chi}_2\sigma_y(\boldsymbol{\sigma}\times\hat{\mathbf{k}})_m\chi_1\right](\chi_m^\dagger\sigma_y\tilde{\chi}_3^\dagger).\end{aligned} \tag{18}$$

The dependence of the differential cross-section on the polarizations of the colliding nucleons can be written in a standard form, which holds for S-wave production:

$$\begin{aligned}&\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(-)}(\mathbf{pp}\to\Sigma^+\Theta^+)=\\ &\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0\left[1+\mathcal{A}_{1\Sigma}^{(-)}\mathbf{P}_1\cdot\mathbf{P}_2+\mathcal{A}_{2\Sigma}^{(-)}\hat{\mathbf{k}}\cdot\mathbf{P}_1\hat{\mathbf{k}}\cdot\mathbf{P}_2\right]\end{aligned} \tag{19}$$

with the following formulas for $\mathcal{A}_{1,2\Sigma}$:

$$\begin{aligned}D_{\Sigma}^{(-)}\mathcal{A}_{1\Sigma}^{(-)}&=|f_{1\Sigma}^{(+)}|^2,\\ D_{\Sigma}^{(-)}\mathcal{A}_{2\Sigma}^{(-)}&=2\left[-|f_{1\Sigma}^{(+)}|^2+|f_{2\Sigma}^{(+)}|^2+|f_{3\Sigma}^{(+)}|^2\right],\end{aligned} \tag{20}$$

with

$$D_{\Sigma}^{(-)}=|f_{1\Sigma}^{(-)}|^2+2\left(|f_{2\Sigma}^{(-)}|^2+|f_{3\Sigma}^{(-)}|^2\right),$$

i.e.

$$A_{xx}^{(-)}(\Sigma)=A_{yy}^{(-)}(\Sigma)=\mathcal{A}_{1\Sigma}^{(-)}\geq 0. \tag{21}$$

The polarization transfer coefficients can be expressed as a function of the amplitudes as

$$\mathbf{P}_{\Sigma}^{(-)}=p_{1\Sigma}^{(-)}\mathbf{P}+p_{2\Sigma}^{(-)}\hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\mathbf{P}), \tag{22}$$

$$D_{\Sigma}^{(-)}p_{1\Sigma}^{(-)}=\mathrm{Re}f_{1\Sigma}^{(-)}(2f_{2\Sigma}^{(-)}+f_{3\Sigma}^{(-)})^*, \tag{23}$$

$$\begin{aligned}D_{\Sigma}^{(-)}p_{2\Sigma}^{(-)}&=2|f_{2\Sigma}^{(-)}|^2-|f_{3\Sigma}^{(-)}|^2-\mathrm{Re}\ f_{1\Sigma}^{(-)}f_{3\Sigma}^{(-)*}\\ &-2\mathrm{Re}\left(2f_{1\Sigma}^{(-)}-f_{3\Sigma}^{(-)}\right)f_{2\Sigma}^{(-)*}.\end{aligned} \tag{24}$$

Let us evaluate these observables again in the framework of the above-mentioned K-exchange model for $p+p\to\Sigma^++\Theta^+$. Two Feynman diagrams, in analogy with those of fig. 1 generate the following relation between the partial amplitudes $f_{i\Sigma}^{(-)}$:

$$f_{2\Sigma}^{(-)}=0,\ f_{1\Sigma}^{(-)}=-f_{3\Sigma}^{(-)}, \tag{25}$$

again independent of coupling constants and phenomenological form factors. This allows to find

$$\begin{aligned}A_{xx}^{(-)}(\Sigma)&=A_{yy}^{(-)}(\Sigma)=A_{zz}^{(-)}(\Sigma)=\frac{1}{3},\\ D_{xx}^{(-)}(\Sigma)&=D_{yy}^{(-)}(\Sigma)=D_{zz}(\Sigma)^{(-)}=-\frac{1}{3}.\end{aligned} \tag{26}$$

3 The reaction $\mathbf{N}+\mathbf{N}\to\mathbf{Y}+\mathbf{\Theta}^+(\mathbf{j}^{\mathsf{P}})$, any $\mathbf{j}^{\mathsf{P}}$

In this section, we consider the case of any j^P for the Θ^+-hyperon, produced in the reactions $N+N\to Y+\Theta^+$, $Y=\Lambda$- or Σ^+-hyperon.

At the reaction threshold, the polarization properties of the $\Theta^+(j^P)$ can be described by the $\mathbf{q}\to 0$ limit of the corresponding Rarita-Schwinger spinor ($\mathbf{q}$ is the Θ^+ three-momentum), more exactly by the following two-component spinor $\chi_{a_1,\ldots,a_n}$, $n=j-1/2, j\geq 3/2$, with definite number of vector indices. Such spinor has to satisfy the conditions

$$\begin{aligned}\sigma_a\chi_{aa_2a_3,\ldots,a_n}&=0,\ \chi_{aaa_3,\ldots,a_n}=0,\\ \chi_{a_1a_2,\ldots,a_n}&=\chi_{a_2a_1,\ldots,a_n},\end{aligned} \tag{27}$$

and this last property guarantees the symmetry with respect to the interchange of any pair of vector indices a_i, $i=1,\ldots,n$.

Evidently, such construction has $2j+1$ independent components, as must be the case for a particle with spin j.

We will apply this general formalism, for the description of the polarization properties of a fermion with spin j, to the process $p+p\to\Sigma^++\Theta^+$ in threshold conditions. The process $n+p\to\Lambda^0+\Theta^+$ can be considered in a similar way.

3.1 The reaction $p + p \to \Sigma^+ + \Theta^+(j^+)$

In this section we consider the case when the P-parity of Θ^+ is positive. The selection rules allow only one partial transition:

$$S_i = 0,\quad \ell_i = \text{even} \ \to \mathcal{J} = \ell \to S_f = \ell_i,\quad \ell_f = 0, \quad (28)$$

with the following relation between ℓ_i and j:

$$\ell_i = j - \frac{1}{2},\ \text{if } P(\Theta) = (-1)^{j-1/2},$$
$$i.e.\ \left(j^P = \frac{1}{2}^+,\ \frac{5}{2}^+,\ \frac{9}{2}^+ \ \ldots\right),$$
$$\ell_i = j + \frac{1}{2},\ \text{if } P(\Theta) = (-1)^{j+1/2}$$
$$i.e.\ \left(\ j^P = \frac{3}{2}^+,\ \frac{7}{2}^+,\ \frac{11}{2}^+ \ \ldots\right). \quad (29)$$

The first case corresponds to natural (n) parity and the second to unnatural (u) parity.

The corresponding matrix element can be written as follows:

$$\mathcal{M}^{(+)} = f^{(+)}(j)(\tilde{\chi}_2\sigma_y\chi_1)(\chi_4^\dagger\sigma_y\tilde{\chi}_3^\dagger), \quad (30)$$

with

$\chi_4 = \chi_{a_1a_2,\ldots,a_n}\hat{k}_{a_1}\hat{k}_{a_2},\ldots,\hat{k}_{a_n}$, for natural P-parity,
$\chi_4 = \boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\chi_{a_1a_2,\ldots,a_n}\hat{k}_{a_1}\hat{k}_{a_2},\ldots,\hat{k}_{a_n}$, for unnatural P-parity,

where $f^{(+)}(j)$ is the partial amplitude, depending on the Θ spin j.

Independently of the value of the Θ^+ spin j, from the matrix element (30) one derives the general form for all double-spin polarization observables:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(+)}(j) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 (1 - \mathbf{P}_1\cdot\mathbf{P}_2),$$
$$D_{xx}^{(+)}(j) = D_{yy}^{(+)}(j) = D_{zz}^{(+)}(j) = 0, \quad (31)$$

due to the "closed" initial singlet *pp*-state.

3.2 The reaction $p + p \to \Sigma^+ + \Theta^+(j^-)$

The case of negative P-parity must be treated separately, for natural- and unnatural-parity states.

3.2.1 Natural P-parity

For $j^P = \frac{3}{2}^-,\ \frac{7}{2}^-, \ldots$ the following partial transitions are allowed:

$$S_i = 1,\quad \ell_i = j - \frac{1}{2} \to \mathcal{J} = j - \frac{1}{2} \to S_f = j - \frac{1}{2},$$
$$\to \mathcal{J} = j + \frac{1}{2} \to S_f = j + \frac{1}{2}, \quad (32)$$
$$S_i = 1,\quad \ell_i = j + \frac{3}{2} \to \mathcal{J} = j + \frac{1}{2} \to S_f = j + \frac{1}{2},$$

with the corresponding spin structure of the matrix element:

$$\mathcal{M}_n^{(-)}(j) = f_{1n}^{(-)}(j)(\tilde{\chi}_2\sigma_y\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\chi_1)(\chi_4^\dagger\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\sigma_y\tilde{\chi}_3^\dagger)$$
$$+if_{2n}^{(-)}(j)\left[\tilde{\chi}_2\sigma_y(\boldsymbol{\sigma}\times\hat{\mathbf{k}})_m\chi_1\right](\chi_{4m}^\dagger\sigma_y\tilde{\chi}_3^\dagger), \quad (33)$$
$$+f_{3n}^{(-)}(j)(\tilde{\chi}_2\sigma_y\sigma_m\chi_1)\left[\chi_4^\dagger(\sigma_m - \hat{k}_m\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\sigma_y\tilde{\chi}_3^\dagger\right],$$

where $f_{in}^{(-)}(j)$, $i = 1$–3, are the independent partial amplitudes and $\chi_4 \equiv \chi_{a_1a_2,\ldots,a_n}\hat{k}_{a_1}\hat{k}_{a_2},\ldots,\hat{k}_{a_n}$, $\chi_{4m} \equiv \chi_{ma_2,\ldots,a_n}\hat{k}_{a_2},\ldots,\hat{k}_{a_n}$.

3.2.2 Unnatural P-parity

The case $j^P = \frac{1}{2}^-,\ \frac{5}{2}^-, \ldots$ generates the following set of triplet transitions:

$$S_i = 1,\quad \ell_i = j - \frac{3}{2} \to \mathcal{J} = j - \frac{1}{2} \to S_f = j - \frac{1}{2},$$
$$S_i = 1,\quad \ell_i = j + \frac{1}{2} \to \mathcal{J} = j - \frac{1}{2} \to S_f = j - \frac{1}{2}, \quad (34)$$
$$\to \mathcal{J} = j + \frac{1}{2} \to S_f = j + \frac{1}{2},$$

with the corresponding matrix element:

$$\mathcal{M}_u^{(-)}(j) = f_{1u}^{(-)}(j)(\tilde{\chi}_2\sigma_y\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}\chi_1)(\chi_4^\dagger\sigma_y\tilde{\chi}_3^\dagger)$$
$$+if_{2u}^{(-)}(j)\left[\tilde{\chi}_2(\boldsymbol{\sigma}\times\hat{\mathbf{k}})_m\chi_1\right](\chi_4^\dagger\sigma_m\sigma_y\tilde{\chi}_3^\dagger) \quad (35)$$
$$+f_{3u}^{(-)}(j)\left[\tilde{\chi}_2(\sigma_m - \hat{k}_m\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\chi_1\right](\chi_{4m}^\dagger\sigma_y\tilde{\chi}_3^\dagger).$$

Equations (33) and (35) allow to express the coefficients $\mathcal{A}_i^{(-)}(j)$, $i = 1, 2$, in terms of the partial amplitudes $f_{in}^{(-)}(j)$, or $(f_{iu}^{(-)}(j))$:

$$\mathcal{A}_1^{(-)}(j) = \frac{|f_1^{(-)}(j)|^2}{|f_1^{(-)}(j)|^2 + 2(|f_2^{(-)}(j)|^2 + |f_3^{(-)}(j)|^2},$$
$$3\mathcal{A}_1^{(-)}(j) + \mathcal{A}_2^{(-)}(j) = 1, \quad (36)$$

(the indices u and n are not indicated in these formulas).

One can see that for any j, the negative parity of the Θ-hyperon results in

$$A_{xx}^{(-)}(j) = A_{yy}^{(-)}(j) \geq 0. \quad (37)$$

In the general case, the numerical value of the asymmetries depend on j, being, however, positive. This holds for the process $p + p \to \Sigma^+ + \Theta^+$.

Note, finally, that in the K-meson exchange model (which is taken here for illustrative purposes), one can find

$$f_2^{(-)}(j) = 0,\quad f_1^{(-)}(j) = \pm f_3^{(-)}(j) \neq 0, \quad (38)$$

where the sign $\pm$ corresponds to natural or unnatural P-parity, with the following universal result for any j:

$$A_{xx}^{(-)}(j) = A_{yy}^{(-)}(j) = A_{zz}^{(-)}(j) = \frac{1}{3}, \quad (39)$$

i.e. in this model the asymmetries do not depend on j.

4 Conclusions

We calculated double-spin polarization observables in the simplest processes of Θ^+ production in NN collisions, $n+p \to \Lambda^0+\Theta^+$ and $p+p \to \Sigma^+ + \Theta^+$. We proved that the spin correlation coefficients A_{xx} and A_{yy} (in the collisions of transversally polarized nucleons) and the polarization transfer coefficient D_{yy} (characterizing the transversal polarization of the final hyperon, Σ^+ or Λ^0 self-analyzing particles, emitted in the collision of a polarized (unpolarized) nucleon beam with unpolarized (polarized) nucleon target) can be considered as model-independent filters for the determination of the P-parity of the Θ^+-hyperon, whatever value takes its spin.

We found that the spin structure of the matrix element is essentially different for different parities. Whereas only the singlet amplitude is present in $p + p \to \Sigma^+ + \Theta^+$, for positive parity and any spin, for the process $n + p \to \Lambda^0 + \Theta^+$ the singlet amplitude is associated to a negative parity (for any spin).

In the same formalism, we established that the process $p + p \to \Sigma^+ + \Theta^+(j^-)$ is characterized by three triplet amplitudes, for any j, $j \geq 3/2$. Similar triplet amplitudes define the spin structure of the threshold matrix element for $n + p \to \Lambda^0 + \Theta^+$, in case of positive Θ^+ parity.

We stress once more that the functional forms of the spin structure and polarization phenomena in both reactions $n + p \to \Lambda^0 + \Theta^+$ and $p + p \to \Sigma^+ + \Theta^+$, have been done in a model-independent way, using only the general selection rules which hold for strong interaction, as conservation of total angular momentum, isotopic invariance, Pauli principle.

This allowed us to generalize the previously proposed methods for the Θ^+ parity determination in the case of $j = 1/2$ to any value of the spin. The main result of this work can be formulated as follows: the sign of the asymmetry A_{yy} (which is different for $p + p \to \Sigma^+ + \Theta^+$ and $n + p \to \Lambda^0 + \Theta^+$) is uniquely related to the Θ^+ parity, independently of its spin j. This is also correct for the polarization transfer coefficient D_{yy}: a value equal or different from zero of this coefficient is an unambiguous signature of the discussed P-parity.

Numerical estimations of these polarization observables were done using a simple but realistic dynamical model, for the considered reactions, based on K-meson exchange. In the framework of this model, the relations among the threshold partial amplitudes are independent of the coupling constants for the vertices of the considered Feynman diagrams and of the parametrization of the phenomenological form factors, quantities which enter in the calculation of the differential cross-section. Polarization phenomena do not depend on these ingredients of the model, but are more sensitive to more general properties of the reaction mechanism, such as the quantum numbers of the exchanged particles.

The polarization phenomena discussed here are T-even, so they do not vanish even in the framework of a simple model, with real amplitudes. Moreover they are not very sensitive to the effects of initial- and final-state interaction.

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